

1.

$$\begin{vmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix} \stackrel{(-1)}{=} \begin{vmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 0 & 1 \end{vmatrix} \stackrel{(-1)}{=} \begin{vmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \stackrel{1.1.2.2}{=} \underline{\underline{4}}$$

$$\begin{vmatrix} 3 & 3 & -1 & 1 \\ -1 & 2 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 2 & 1 & -2 & 2 \end{vmatrix} \stackrel{(-1)}{=} - \begin{vmatrix} 1 & 2 & 1 & -1 \\ -1 & 2 & 1 & 0 \\ 3 & 3 & -1 & 1 \\ 2 & 1 & -2 & 2 \end{vmatrix} \stackrel{(-1)}{=} \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 4 & 2 & -1 \\ 0 & -3 & -4 & 4 \\ 0 & -3 & -4 & 4 \end{vmatrix} \stackrel{(-1)}{=}$$

$$= \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 4 & 2 & -1 \\ 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

2.

$$= \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & -3 & -4 & 4 \\ 0 & -3 & -4 & 4 \end{vmatrix} \begin{matrix} (3) \\ \downarrow \\ \downarrow \end{matrix}$$

$$= \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & -10 & 13 \\ 0 & 0 & -10 & 13 \end{vmatrix} \begin{matrix} \\ \\ -1 \\ \downarrow \end{matrix} = \begin{vmatrix} 1 & & & \\ & 1 & & \\ & 0 & -10 & 13 \\ & 0 & 0 & 0 \end{vmatrix} =$$

$$= 1 \cdot 1 \cdot (-10) \cdot 0 = 0$$

3.

$$A = \begin{pmatrix} 0 & -3 & -3 \\ 2 & 5 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

Charakteristická rovnice

$$\det(A - \lambda E) = 0$$

$$\begin{vmatrix} -\lambda & -3 & -3 \\ 2 & 5-\lambda & -3 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \cdot \begin{vmatrix} -\lambda & -3 \\ 2 & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \cdot [-\lambda(5-\lambda) - 2(-3)] = 0$$

$$(1-\lambda) \cdot [\lambda^2 - 5\lambda + 6] = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = 3$$

$$\lambda_3 = 2$$

$$D = 25 - 24 = 1$$

$$\lambda_{2,3} = \frac{5 \pm 1}{2}$$

$$4. \quad (1-\lambda)(\lambda-3)(\lambda-2)=0$$

$$\lambda_1 = 1$$

$$A - 1 \cdot E = 0$$

$$\left(\begin{array}{ccc|c} -1 & -3 & -3 & 0 \\ 2 & 4 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} (2) \\ \downarrow \end{array} \sim \left(\begin{array}{ccc|c} -1 & -3 & -3 & 0 \\ 0 & -2 & -9 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$-x_1 - 3x_2 - 3x_3 = 0$$

$$-2x_2 - 9x_3 = 0$$

$$-x_1 + 27 - 6 = 0$$

$$x_1 = 21$$

$$x_3 = 2$$

$$-2x_2 - 18 = 0$$

$$x_2 = -9$$

$$v^1 = \begin{pmatrix} 21 \\ -9 \\ 2 \end{pmatrix}$$

$$5. \lambda_2 = 3$$

$$A - 3E = 0$$

$$\begin{pmatrix} -3 & -3 & -3 & | & 0 \\ 2 & 2 & -3 & | & 0 \\ 0 & 0 & -2 & | & 0 \end{pmatrix} \xrightarrow[-2]{-\frac{1}{3}} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & -5 & | & 0 \\ 0 & 0 & -2 & | & 0 \end{pmatrix} \xrightarrow{(-\frac{1}{5})} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + x_2 + x_3 = 0$$

$$x_3 = 0$$

$$x_1 + x_2 = 0$$

$$x_2 = 1$$

$$x_1 = -1$$

$$v^2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$6. \lambda_3 = 2$$

$$A - 2E = 0$$

$$\left(\begin{array}{ccc|c} -2 & -3 & -3 & 0 \\ 2 & 3 & -3 & 0 \\ 0 & 0 & -1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} -2 & -3 & -3 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & -1 & 0 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} -2 & -3 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{aligned} -2x_1 - 3x_2 - 3x_3 &= 0 \\ x_3 &= 0 \end{aligned}$$

$$v^3 = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$$

$$-2x_1 - 3x_2 = 0$$

$$x_2 = 2$$

$$-2x_1 - 6 = 0$$

$$x_1 = -3$$

$$7. A = \begin{pmatrix} 1 & 2 & 2 & -1 \\ 2 & 1 & -2 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 5 & 4 \end{pmatrix}$$

Charakteristická rovnice

$$\det(A - \lambda E) = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 2 & -1 \\ 2 & 1-\lambda & -2 & 1 \\ 0 & 0 & 2-\lambda & 3 \\ 0 & 0 & 5 & 4-\lambda \end{vmatrix} = 0$$

$$8. (1-\lambda) \cdot \begin{vmatrix} 1-\lambda & -2 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 5 & 4-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 & -1 \\ 0 & 2-\lambda & 3 \\ 0 & 5 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \left[(1-\lambda) \begin{vmatrix} 2-\lambda & 3 \\ 5 & 4-\lambda \end{vmatrix} \right] - 2 \left[2 \begin{vmatrix} 2-\lambda & 3 \\ 5 & 4-\lambda \end{vmatrix} \right] = 0$$

$$(1-\lambda)^2 [(2-\lambda)(4-\lambda) - 15] - 4 [(2-\lambda)(4-\lambda) - 15] = 0$$

$$[(2-2)(4-\lambda) - 15] \cdot [(1-\lambda)^2 - 4] = 0$$

$$[8 - 2\lambda - 4\lambda + \lambda^2 - 15] \cdot [1 - 2\lambda + \lambda^2 - 4] = 0$$

$$(\lambda^2 - 6\lambda - 7) \cdot [\lambda^2 - 2\lambda - 3] = 0$$

$$D = 36 + 28 = 64$$

$$D = 4 + 12 = 16$$

9. $\lambda_{1,2} = \frac{6 \pm 8}{2} \begin{matrix} / 7 \\ \backslash -1 \end{matrix}$

$\lambda_{3,4} = \frac{2 \pm 4}{2} \begin{matrix} / 3 \\ \backslash -1 \end{matrix}$

 $\lambda_{1,2} = -1$

$\lambda_3 = 3$

$\lambda_4 = 7$

$$v^1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

10. $\lambda_{1,2} = -1$

$$A - (-1)E = 0$$

$$\begin{pmatrix} 2 & 2 & 2 & -1 & | & 0 \\ 2 & 2 & -2 & 1 & | & 0 \\ 0 & 0 & 3 & 3 & | & 0 \\ 0 & 0 & 5 & 5 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 2 & -1 & | & 0 \\ 0 & 0 & -4 & 2 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 2 & 2 & 2 & -1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & -2 & 1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 2 & -1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & 3 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_4 = 0$$

$$x_3 = 0$$

$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 - x_4 &= 0 \\ 2x_1 + 2x_2 &= 0 \\ x_2 &= 1 \\ x_1 &= -1 \\ x_3 + x_4 &= 0 \\ 3x_4 &= 0 \end{aligned}$$

$$11. \lambda_3 = 3$$

$$A - 3E = 0$$

$$\left(\begin{array}{cccc|c} -2 & 2 & 2 & -1 & 0 \\ 2 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 5 & 1 & 0 \end{array} \right) \xrightarrow{(5)} \left(\begin{array}{cccc|c} -2 & 2 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 16 & 0 \end{array} \right)$$

$$-2x_1 + 2x_2 + 2x_3 - x_4 = 0$$

$$-x_3 + 3x_4 = 0$$

$$16x_4 = 0$$

$$x_4 = 0$$

$$x_3 = 0$$

$$-2x_1 + 2x_2 = 0$$

$$x_2 = 1$$

$$x_1 = 1$$

$$v^3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$12. \lambda_n = 7$$

$$A - 7E = 0$$

$$\left(\begin{array}{cccc|c} -6 & 2 & 2 & -1 & 0 \\ 2 & -6 & -2 & 1 & 0 \\ 0 & 0 & -5 & 3 & 0 \\ 0 & 0 & 5 & -3 & 0 \end{array} \right) \sim$$

$$\left(\begin{array}{cccc|c} 2 & -6 & -2 & 1 & 0 \\ -6 & 2 & 2 & -1 & 0 \\ 0 & 0 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 2 & -6 & -2 & 1 & 0 \\ 0 & -16 & -4 & 2 & 0 \\ 0 & 0 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \sim$$

$$\left(\begin{array}{cccc|c} 2 & -6 & -2 & 1 & 0 \\ 0 & -8 & -2 & 1 & 0 \\ 0 & 0 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$2x_1 - 6x_2 - 2x_3 + x_4 = 0$$

$$-8x_2 - 2x_3 + x_4 = 0$$

$$-5x_3 + 3x_4 = 0$$

$$x_4 = 5$$

$$-5x_3 + 15 = 0$$

$$x_3 = 3$$

13.

$$-8x_2 - 6 + 5 = 0$$

$$-8x_2 = 1$$

$$x_2 = -\frac{1}{8}$$

$$2x_1 + \frac{6}{8} - 6 + 5 = 0$$

$$V^4 = \begin{pmatrix} \frac{1}{8} \\ -\frac{1}{8} \\ 3 \\ 5 \end{pmatrix}$$

$$2x_1 = \frac{2}{8} = \frac{1}{4}$$

$$x_1 = \frac{1}{8}$$

$$V^5 = \begin{pmatrix} 1 \\ -1 \\ 25 \\ 40 \end{pmatrix}$$

14.

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & -2 \\ 4 & 2 & 0 \end{pmatrix}$$

Charakteristická rovnice.

$$\det(A - \lambda E) = 0$$

$$\begin{vmatrix} -\lambda & 0 & 0 \\ 5 & -\lambda & -2 \\ 4 & 2 & -\lambda \end{vmatrix} = 0$$

$$-\lambda \cdot \begin{vmatrix} -\lambda & -2 \\ 2 & -\lambda \end{vmatrix} = 0$$

15.

$$-2[\lambda^2 + 4] = 0$$

$$\lambda_1 = 0$$

$$\lambda^2 + 4 = 0$$

$$D = 0 - 16 = -16$$

$$\begin{aligned}\lambda_{2,3} &= \frac{\pm\sqrt{-16}}{2} = \\ &= \frac{\pm 4i}{2} = \pm 2i\end{aligned}$$

$$16. \lambda_1 = 0$$

$$A - 0 \cdot E = 0$$

$$\begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 5 & 0 & -2 & | & 0 \\ 4 & 2 & 0 & | & 0 \end{pmatrix} \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_2 \leftrightarrow R_3}} \begin{pmatrix} 1 & -2 & -2 & | & 0 \\ 4 & 2 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & -2 & | & 0 \\ 0 & 10 & 8 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + 8 - 10 = 0$$

$$x_1 = 2$$

$$v^1 = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$$

$$\begin{aligned} x_1 - 2x_2 - 2x_3 &= 0 \\ 5x_2 + 4x_3 &= 0 \end{aligned}$$

$$x_3 = 5$$

$$5x_2 + 20 = 0$$

$$x_2 = -4$$

$$17. \lambda_2 = 2i$$

$$A - 2iE = 0$$

~~$-2ix_1 = 0$~~
 ~~$5x_1$~~

$$\begin{pmatrix} -2i & 0 & 0 & | & 0 \\ 5 & -2i & -2 & | & 0 \\ 4 & 2 & -2i & | & 0 \end{pmatrix} \xrightarrow{2i} \begin{pmatrix} 4 & 0 & 0 & | & 0 \\ 5 & -2i & -2 & | & 0 \\ 4 & 2 & -2i & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & | & 0 \\ 5 & -2i & -2 & | & 0 \\ 4 & 2 & -2i & | & 0 \end{pmatrix} \xrightarrow{\substack{-5 \\ -4}} \begin{pmatrix} 1 & 0 & 0 & | & 0 \\ 0 & -2i & -2 & | & 0 \\ 0 & 2 & -2i & | & 0 \end{pmatrix} \xrightarrow{2i}$$

18.

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 4 & -4i & 0 \\ 0 & 2 & -2i & 0 \end{array} \right) \xrightarrow{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -i & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1$$

$$= 0$$

$$x_2 - ix_3 = 0$$

$$v_2 = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}$$

$$x_1 = 0$$

$$x_3 = 1$$

$$x_2 - i = 0$$

$$x_2 = i$$

$$19. \lambda_3 = -2i$$

$$A + 2iE = 0$$

$$\left(\begin{array}{ccc|c} 2i & 0 & 0 & 0 \\ 5 & 2i & -2 & 0 \\ 4 & 2 & 2i & 0 \end{array} \right) \xrightarrow{2i} \left(\begin{array}{ccc|c} -4 & 0 & 0 & 0 \\ 5 & 2i & -2 & 0 \\ 4 & 2 & 2i & 0 \end{array} \right) \xrightarrow{\cdot (-\frac{1}{4})}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 5 & 2i & -2 & 0 \\ 4 & 2 & 2i & 0 \end{array} \right) \xrightarrow{\substack{(-5) \text{ } (-4) \\ \swarrow \searrow}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2i & -2 & 0 \\ 0 & 2 & 2i & 0 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & i & -1 & 0 \\ 0 & 1 & i & 0 \end{array} \right) \xrightarrow{i} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & -i & 0 \\ 0 & 1 & i & 0 \end{array} \right) \downarrow$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & -i & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

20.

$$x_1$$

$$= 0$$

$$x_1 = 0$$

$$-x_2 - ix_3 = 0$$

$$x_3 = 1$$

$$v^3 = \begin{pmatrix} 0 \\ -i \\ 1 \end{pmatrix}$$

$$-x_2 - i = 0$$

$$x_2 = -i$$