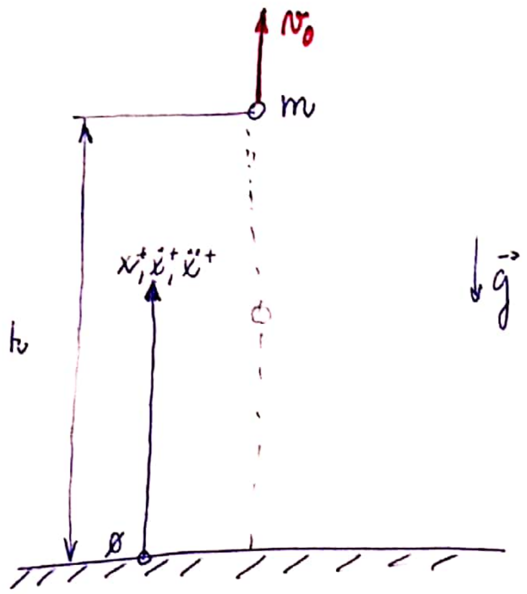
Dynamics of particle

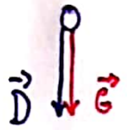
Forced fall

Given: m ... mass of particle
h ... initial position
v_0 ... initial velocity

Task:
$v(t)$ ... velocity in time
$x(t)$ ... displacement —"—



Free body diagram
(in general position)



$\vec{D} = -m\vec{a}$

general eqn. of dynamic equilibrium

$$\sum_{(i)} \vec{F}_i + \vec{D} = \vec{\emptyset}$$

$$\vec{G} + \vec{D} = \vec{\emptyset}$$

scalar form

(1) $D + G = \emptyset$

(1) $\cancel{m}\cdot a + \cancel{m}g = \emptyset$

$$\underline{a = -g}$$

force specification

$D = m \cdot a$

$G = mg$

1) Velocity of the particle

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = -g \quad /dt$$

$$\int_{v_0}^{v(t)} dv = -g\int_0^t dt$$

$$v(t) - v_0 = -gt$$

$$\underline{\underline{v(t) = v_0 - gt}}$$

2) Displacement of the particle

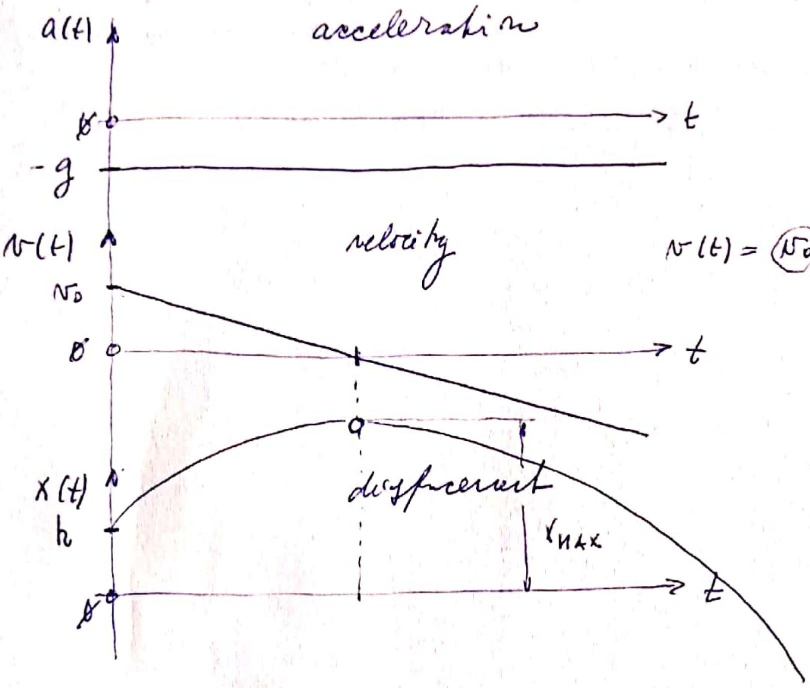
$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = v_0 - gt$$

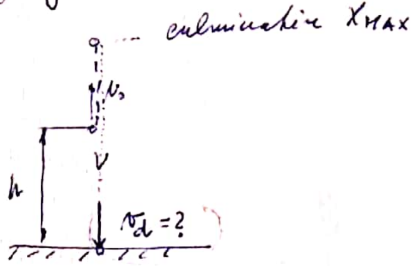
$$\int_{h}^{x(t)} dx = \int_{\emptyset}^{t} (v_0 - gt)\, dt$$

$$x(t) - h = v_0 t - \frac{1}{2} g t^2$$

$$x(t) = h + v_0 t - \frac{1}{2} g t^2$$



$v(t) = v_0 - gt$



$v_d = ?$ when particle drops down

(2) $v(t) = v_0 - gt$ $\qquad x(t = t_d) = \emptyset$

(3) $x(t) = h + v_0 t - \frac{1}{2} g t^2$ $\qquad v(t = t_d) = v_d$

$$\left.\begin{aligned} (2)\quad & v_d = v_0 - g t_d \\ (3)\quad & \emptyset = h + v_0 t_d - \frac{1}{2} g t_d^2 \end{aligned}\right\} \begin{array}{l} \text{2 eqns} \\ \text{2 unknowns} \\ (t_d, v_d) \end{array}$$

Scanned with CamScanner

definition:

$$a = \frac{dv}{dt}\cdot\underbrace{\frac{dx}{dx}}_{1} = \frac{dv}{dx}\cdot\underbrace{\frac{dx}{dt}}_{v}$$

$$\underline{a = v\frac{dv}{dx}}$$

$$v\frac{dv}{dx} = -g \quad / dx$$

$$\int_{v_0}^{v(x)} v\,dv = -g\int_{h}^{x} dx$$

$$\left[\frac{1}{2}v^2\right]_{v_0}^{v(x)} = -g\,(x-h)$$

$$\frac{1}{2}\left(v^2(x)-v_0^2\right) = -g\,(x-h)$$

$$v(x) = \pm\sqrt{v_0^2-2g\,(x-h)}$$

when $x = 0$

$$v_d = \pm\sqrt{v_0^2+2gh}$$