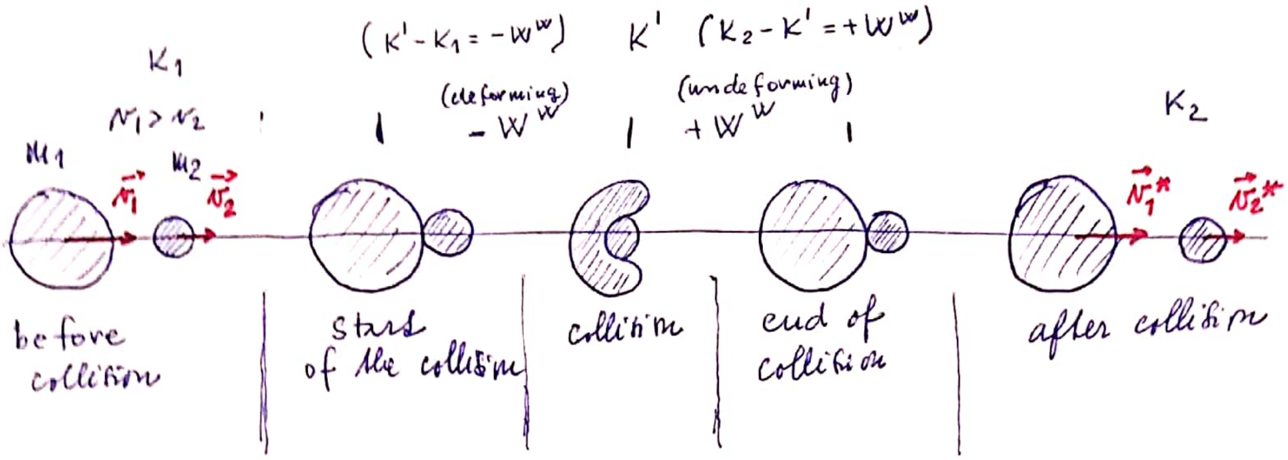
PERFECTLY ELASTIC COLLISION



m_1, m_2 ... mass of particles
v_1, v_2 ... velocity of particles bebore collision
v_1^*, v_2^* ... —— " —— after collision

K_1 ... kinetic energy of system of particles before collision
K_2 ... —— " —— after —"—

$$K_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$
$$K_2 = \frac{1}{2} m_1 v_1^{*2} + \frac{1}{2} m_2 v_2^{*2}$$

The collision is perfectly ellastic so there is no energy loss.
$\Rightarrow$ $K_1 = K_2$. We use the law of conservation of mechanical energy:

$$(1) \qquad \cancel{\frac{1}{2}} m_1 v_1^2 + \cancel{\frac{1}{2}} m_2 v_2^2 = \cancel{\frac{1}{2}} m_1 v_1^{*2} + \cancel{\frac{1}{2}} m_2 v_2^{*2}$$

$$(1) \qquad \boxed{m_1 (v_1^2 - v_1^{*2}) = m_2 (v_2^{*2} - v_2^2)}$$

Next we can use the law of conservation of momentum becose there is no external force actin on the system. $p_1 = p_2$

$$(2) \qquad m_1 \cdot v_1 + m_2 v_2 = m_1 v_1^* + m_2 v_2^*$$

$$(2) \qquad \boxed{m_1 (v_1 - v_1^*) = m_2 (v_2^* - v_2)}$$

(1) $m_1 (v_1 - v_1^*)(v_1 + v_1^*) = m_2 (v_2^* - v_2)(v_2^* + v_2)$

(2) $m_1 (v_1 - v_1^*) = m_2 (v_2^* - v_2)$

(1) $m_2 \cancel{(v_2^* - v_2)} (v_1 + v_1^*) = m_2 \cancel{(v_2^* - v_2)} (v_2^* + v_2)$

(1) $v_1 + v_1^* = v_2^* + v_2$

(2) $v_1 - v_1^* = \frac{m_2}{m_1} (v_2^* - v_2)$

(1) + (2) $2 v_1 = v_2^* \left(1 + \frac{m_2}{m_1}\right) + v_2 \left(1 - \frac{m_2}{m_1}\right)$

$$v_2^* \frac{m_1 + m_2}{m_1} = 2 v_1 - v_2 \frac{m_2 - m_1}{m_1}$$

$$\boxed{v_2^* = \frac{2 v_1 m_1 - v_2 (m_1 - m_2)}{m_1 + m_2}}$$

(1) $v_1^* = v_2^* + v_2 - v_1 = \dots = \boxed{\frac{2 m_2 v_2 - v_1 (m_2 - m_1)}{m_1 + m_2}}$

(exchange of indices)