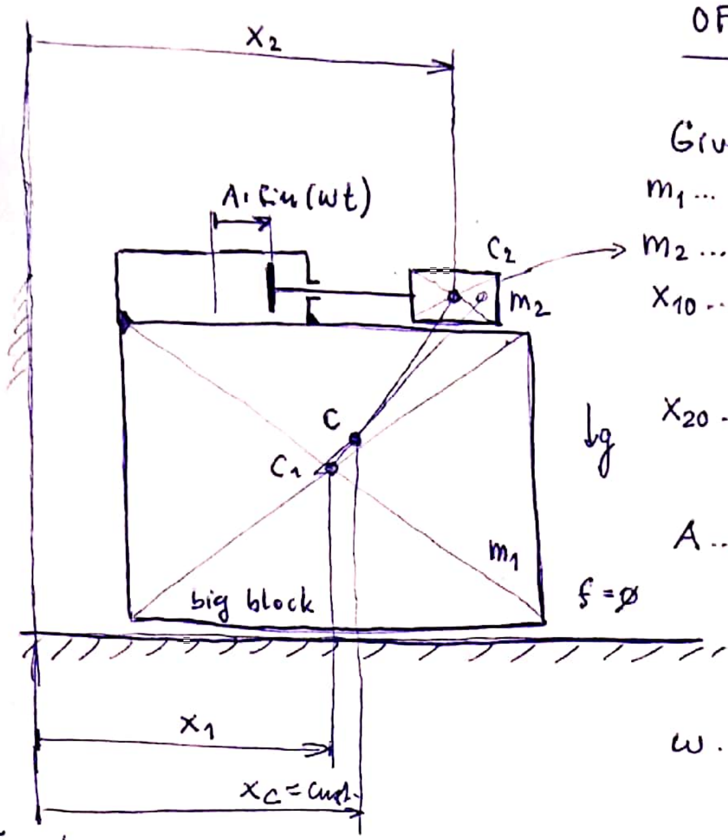
MOTION OF CENTER OF MASS OF SYSTEM OF PARTICLES (example)



Given:

m_1 ... mass of the big block
m_2 ... mass of the small block
x_{10} ... initial position of big block
x_{20} ... initial position of small block
A ... amplitude of motion of small block (relatively to big block)
ω ... angular velocity of relative motion

(ZERO FRICTION)

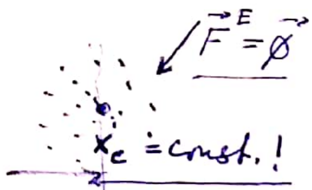
Task:

$x_1(t)$ ... absolute motion of big block
$x_2(t)$ ... —— " —— small —"—

We solve system of two particles (m_1, m_2)
Position center of mass of system of particles at time $t = \emptyset$:

$t=\emptyset$ (1) $\boxed{(m_1+m_2)\cdot x_{C0}} = m_1 x_{10} + m_2 \cdot x_{20}$ $(\Rightarrow x_{C0})$

$t=t$ (2) $\boxed{(m_1+m_2)\, x_C} = m_1 \cdot x_1 + m_2 \cdot x_2$ $(\Rightarrow x_C)$



$x_{C0} = x_C = const.$

(3) $m_1 x_{10} + m_2 x_{20} = m_1 x_1 + m_2 x_2$ (2 unknowns: x_1, x_2)

description of motion of small block (m_1):

(4) $x_2 = x_{20} + A \cdot \sin(\omega t) - (x_{10} - x_1)$

Substituting (4) in (3):

$m_1 x_{10} + m_2 x_{20} = m_1 x_1 + m_2 x_{20} + m_2 A \sin(\omega t) - m_2 x_{10} + m_2 x_1$

$x_1 (m_1 + m_2) = x_{10}(m_1 + m_2) - m_2 A \sin(\omega t)$

$$x_1 = x_{10} - \frac{m_2}{m_1 + m_2} A \sin(\omega t)$$

$$x_2 = x_{20} + \frac{m_1}{m_1 + m_2} A \sin(\omega t)$$