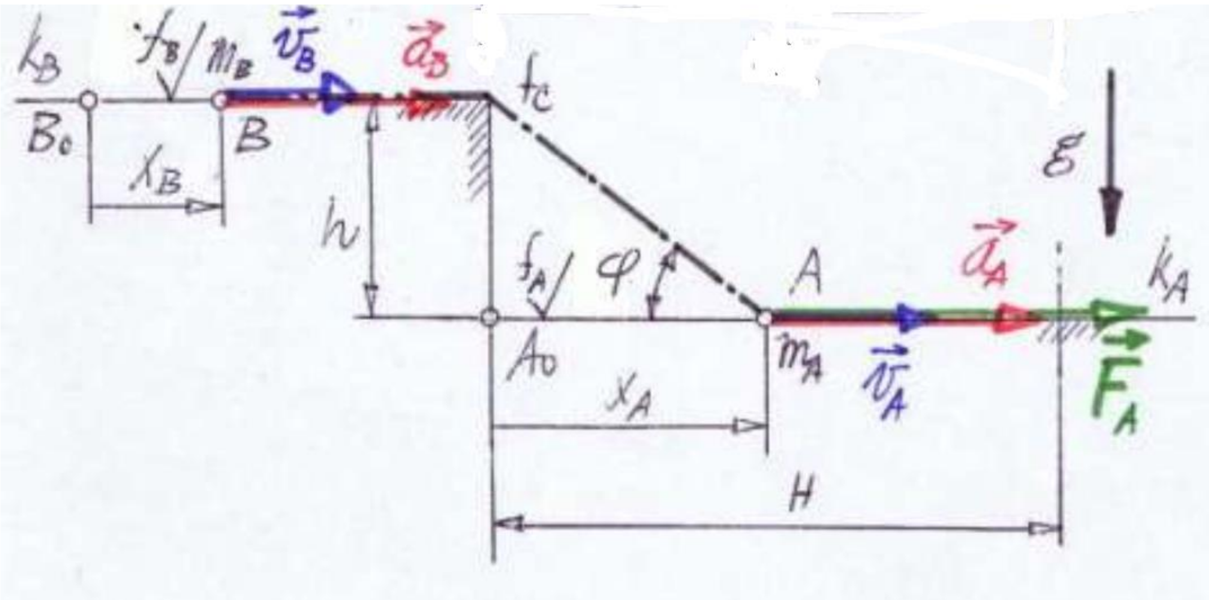
Dynamics of the system of particles

Calculating the dynamics of the system of two particles which is presented in the Figure. Two particles A and B move from the initial positions A_0 and B_0. The trajectories of the particles are line k_A and k_B. The particles are joined together by a rope (with).



Given:

the mass of particles: m_A, m_B,

friction coefficients between the particles and the surfaces when they move: f_A, f_B,

friction coefficients between the rope and the edge C: f_C,

the length of the rope: L,

the relation of the velocity of the particle A and its displacement is presented by the formula: $v_A(x_A) = Vx_A$

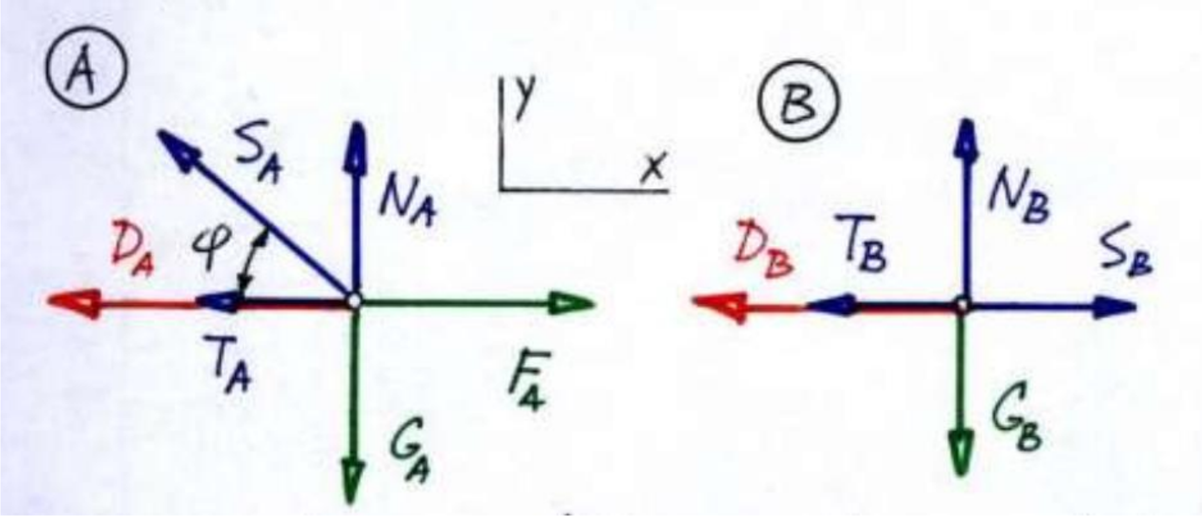
the constants: V, h.

Task:

Write the equations of motion of the system. Find $F_A(x_A)$

Solution:

First, we draw the free body diagram of the particles using the d'Alembert as the Figure



The forces acting on the particle A include

Gravity force $\vec{G}_A$

Normal force $\vec{N}_A$

Applied force $\vec{F}_A$

Friction force $\vec{T}_A$

Tension force $\vec{S}_A$

D'Alember force $\vec{D}_A$

The forces acting on the particle B include

Gravity force $\vec{G}_B$

Normal force $\vec{N}_B$

Friction force $\vec{T}_B$

Tension force $\vec{S}_B$

D'Alember force $\vec{D}_B$

Generally, **the equation of motion** of the particles can be written:

$$\vec{G}_A + \vec{N}_A + \vec{T}_A + \vec{S}_A + \vec{F}_A + \vec{D}_A = \vec{0}$$

$$\vec{G}_B + \vec{N}_B + \vec{T}_B + \vec{S}_B + \vec{D}_A = \vec{0}$$

Then we can write the component equations by the following:

$$x: F_A - T_A - D_A - S_A \cos\varphi = 0 \quad (1)$$

$$y: N_A - G_A + S_A \sin\varphi = 0 \quad (2)$$

$$x: S_B - T_B - D_B = 0 \tag{3}$$
$$y: N_B - G_B = 0 \tag{4}$$

Specification of force components:

Gravity force

$$G_A = m_A g \tag{5}$$
$$G_B = m_B g \tag{6}$$

D'Alember force

$$D_A = m_A a_A \tag{7}$$
$$D_B = m_B a_B \tag{8}$$

Friction force

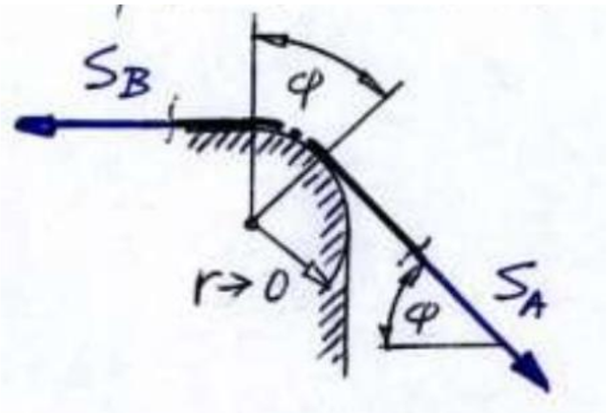
$$T_A = f_A N_A \tag{9}$$
$$T_B = f_B N_B \tag{10}$$

(the normal forces $N_A > 0, N_B > 0$)

The relation of the tension forces can be presented by Euler's equation, as described in the equation below:

$$S_A = S_B e^{f_C \varphi} \tag{11}$$

S_A will depend on the value of S_B, the coefficient of friction between the rope and the eagle (f_C), and the contact angle between the rope and the eagle (φ) given in radians.



Kinematic quantities:

$$tg\varphi = \frac{h}{x_A} \tag{12}$$
$$x_B = \sqrt{x_A^2 + h^2} - h \tag{13}$$

$$v_B = \frac{dx_B}{dt} = \frac{dx_B}{dx_A}\frac{dx_A}{dt} = v_A \frac{dx_B}{dx_A} \quad (14)$$

$$a_A = v_A \frac{dv_A}{dx_A} \quad (15)$$

$$a_B = \frac{dv_B}{dt} = \frac{dv_B}{dx_A}\frac{dx_A}{dt} = v_A \frac{dv_B}{dx_A} \quad (16)$$

Where:

$$\frac{dv_B}{dx_A} = \frac{d}{dx_A}\left(v_A \frac{dx_B}{dx_A}\right) = \frac{dv_A}{dx_A}\frac{dx_B}{dx_A} + v_A \frac{d^2 x_B}{dx_A^2}$$

The system of equations consists of 16 equations from (1) to (16) with 18 variables:

$$F_A, T_A, D_A, S_A, \varphi, N_A, G_A, S_B, T_B, D_B, N_B, G_B, x_A, x_B, v_A, v_B, a_A, a_B$$

In combination with the relation of the velocity of the particle A and its displacement $v_A(x_A) = Vx_A$ as the equation 17[th], we can find the relationship of variables with dependence on x_A.

From (12), we get:

$$\varphi(x_A) = arctg \frac{h}{x_A}$$

From (13), we get:

$$\frac{dx_B}{dx_A} = \frac{x_A}{\sqrt{x_A^2 + h^2}} \quad (17)$$

So:

$$\frac{d^2 x_B}{dx_A^2} = \frac{\sqrt{x_A^2 + h^2} - \frac{x_A}{\sqrt{x_A^2 + h^2}}}{\sqrt{x_A^2 + h^2}} = \frac{h^2}{\left(x_A^2 + h^2\right)^{\frac{3}{2}}}$$

From

$$v_A(x_A) = Vx_A$$

We have:

$$\frac{dv_A}{dx_A} = V$$

Then substitute to the equation (15) and (16), we determine function $a_A(x_A)$, $a_B(x_A)$:

$$a_A = v_A \frac{dv_A}{dx_A} = V^2 x_A$$

$$a_B = v_A \frac{dv_B}{dx_A} = v_A \left(\frac{dv_A}{dx_A} \frac{dx_B}{dx_A} + v_A \frac{d^2 x_B}{dx_A^2} \right) = Vx_A \left(V \frac{x_A}{\sqrt{x_A^2 + h^2}} + Vx_A \frac{h^2}{\left(x_A^2 + h^2 \right)^{\frac{3}{2}}} \right)$$

$$= V^2 x_A^2 \frac{x_A^2 + 2h^2}{\left(x_A^2 + h^2 \right)^{\frac{3}{2}}}$$

Rewrite the equations (1) …(4) in the forms:

$$F_A - f_A N_A - m_A a_A(x_A) - S_A \cos\varphi(x_A) = 0 \quad (18)$$

$$N_A - m_A g + S_A \sin\varphi(x_A) = 0 \quad (19)$$

$$S_B - f_B N_B - m_B a_B(x_B) = 0 \quad (20)$$

$$N_B - m_B g = 0 \quad (21)$$

From (21), we get:

$$N_B = m_B g$$

And from (20), we have:

$$S_B = m_B \left[f_B g + a_B(x_A) \right] \quad (22)$$

From (19), we have:

$$N_A = m_A g - S_A \sin\varphi(x_A)$$

Substitute to (18), we have:

$$F_A - m_A f_A g - m_A a_A(x_A) + S_A \left[f_A \sin\varphi(x_A) - \cos\varphi(x_A) \right] = 0$$

Substitute to (22) to (11), we have:

$$S_A = m_B \left[f_B g + a_B(x_A) \right] e^{f_C \varphi(x_A)}$$

Then we obtain:

$$F_A = m_A \left[f_A g + a_A(x_A) \right] - m_B \left[f_B g + a_B(x_A) \right] e^{f_C \varphi(x_A)} \left[f_A \sin\varphi(x_A) - \cos\varphi(x_A) \right]$$