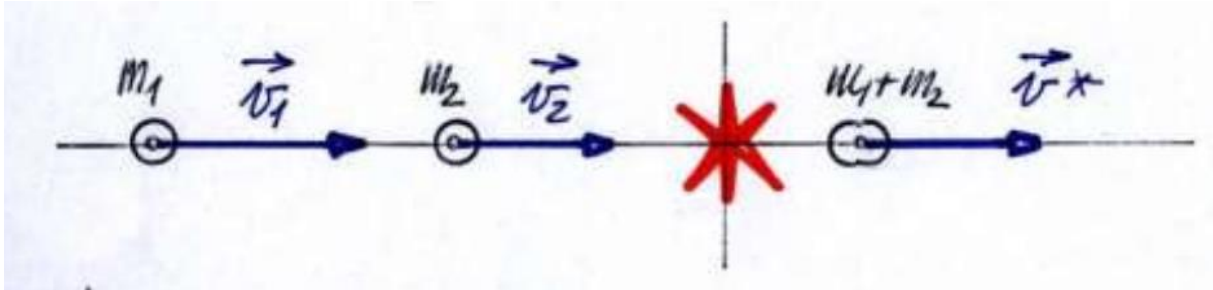


Impact of particles – ideal plastic impact

we consider the collinear motion of two spheres of masses m_1 and m_2 , as Figure, moving with velocities v_1 and v_2 before the impact. After the impact occurs they transformed into one with the sum of masses and velocity v^* .



Given:

The masses of two spheres m_1 and m_2 , velocities v_1 and v_2 before the impact

Task:

Velocity v^* after the impact

The kinetic energy difference after and before the impact.

Solution:

Using the law of conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v^*$$

So we get the velocity after the impact:

$$v^* = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad (1)$$

The kinetic energy of the system of two spheres before the impact is:

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad (2)$$

And after the impact:

$$K^* = \frac{1}{2}(m_1 + m_2)v^{*2} \quad (3)$$

Substitute (1) to (3), we have:

$$K^* = \frac{1}{2}(m_1 + m_2) \left(\frac{m_1v_1 + m_2v_2}{m_1 + m_2} \right)^2 \quad (4)$$

$$K^* = \frac{1}{2} \frac{m_1^2 v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2}{m_1 + m_2}$$

(5)

(6)

So the kinetic energy difference is:

$$\begin{aligned} K^* - K &= \left(\frac{1}{2} \frac{m_1^2 v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2}{m_1 + m_2} \right) - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \\ &= -\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 + v_2)^2 \end{aligned}$$

(7)