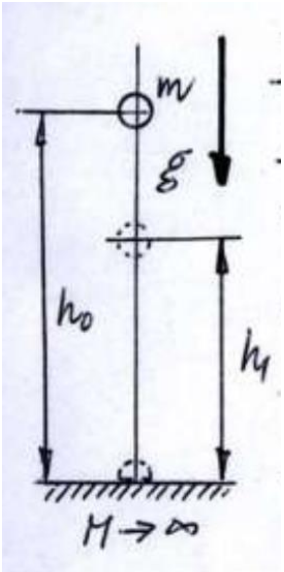
Impact of particles – real impact



Given:

A ball with mass m dropped from the rest at zero initial velocity height h_0 above a heavy horizontal steel plate. Assuming that the steel plate has the mass M -> infinity. After the rebound from the plate, the ball rises to a new height $h_1 < h_0$ where it has zero velocity again.

Task:

Determine the coefficient of restitution e for the ball/the steel plate.

Solution:

Because the working force acting on the ball is the gravity force, so the potential energy of the ball at the initial position is:

$$V_0 = mgh_0 \qquad (1)$$

And the kinetic energy of the ball at the initial position is:

$$K_0 = 0 \qquad (2)$$

Because the initial velocity of the ball is 0.

Before the impact between the ball and the steel plate, the so the potential energy of the ball is:

$$V = 0 \qquad (3)$$

And the kinetic energy of the ball is

$$K = \frac{1}{2}mv^2 \qquad (4)$$

According to the law of conservation of energy, we have:

$$V_0 + K_0 = V + K \quad (5)$$

Or

$$0 + mgh_0 = \frac{1}{2}mv^2 + 0$$

So we get the velocity of the ball before the impact:

$$v = \sqrt{2gh_0} \quad (6)$$

After the rebound from the plate, the ball rises to a new height h2<h0 so the type of impact is real impact, a part of the energy is lost through the impact. In this case, we use the Newton's theorem:

$$\varepsilon = -\frac{v_1^* - v_2^*}{v_1 - v_2} \quad (7)$$

Where:

ε is the coefficient of restitution,

v_1^* is the velocity of the ball after the impact

v_2^* is the velocity of the steel plate after the impact

v_1 is the velocity of the ball before the impact

v_2 is the velocity of the steel plate before the impact

We know, with the assumption that the mass of the plate M -> infinity, its velocity is equal to zero before and after the impact:

$v_2^* = v_2 = 0$

and $v_1 = v = \sqrt{2gh_0}$

so we get:

$$v^* = v_1^* = -\varepsilon v_1 = -\varepsilon\sqrt{2gh_0} \quad (8)$$

Then we continue using the law of conservation of energy for the ball after the impact:

$$\frac{1}{2}mv^{*2} + 0 = 0 + mgh_1 \quad (9)$$

Or

$$\frac{1}{2}m\varepsilon^2 2gh_0 = mgh_1 \tag{10}$$

So we get:

$$\varepsilon = \sqrt{\frac{h_1}{h_0}}$$