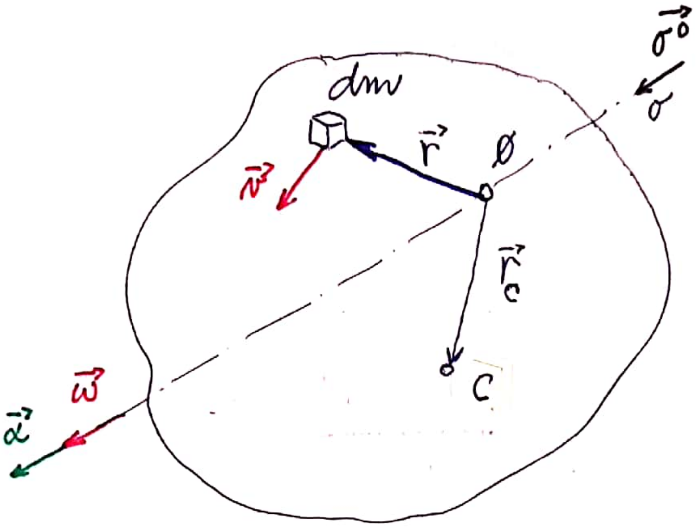
DYNAMICS OF ROTATIONAL MOTION OF BODY



$\vec{o}^0$ ... unit vector of axis
$\vec{\omega}$ ... angular velocity of rotational motion
$\vec{\alpha}$ ... angular acceleration of rotational motion
$\vec{r}$ ... displacement vector
C ... center of mass
$\vec{r}_c$ ... position vector of center of mass (C)

Velocity of general point:

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Velocity of center of mass C:

$$\vec{v}_c = \omega \times \vec{r}_c$$

Momentum of rotating body:

$$\vec{p} = \int_{(m)} \vec{v} \cdot dm = \int_{(m)} \vec{\omega} \times \vec{r}\, dm = \underset{const.}{\vec{\omega}} \times \overbrace{\int_{(m)} \vec{r}\, dm}^{\vec{r}_c \cdot m} = \underbrace{\vec{\omega} \times \vec{r}_c}\cdot m = \underline{\underline{\vec{v}_c \cdot m}}$$

1) Law of change of momentum

$$\underline{\frac{d\vec{p}}{dt} = \vec{F}^E} \qquad \vec{F}^E = \sum_{i=1}^{n} \vec{F}_i^E = \sum_{i=1}^{m} \vec{A}_i + \sum_{j=1}^{o} \vec{R}_j$$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m \cdot \vec{v}_c) = \underset{const.}{m} \cdot \underbrace{\frac{d\vec{v}_c}{dt}}_{\vec{a}_c} = m \cdot \vec{a}_c = m(\vec{a}_{ct} + \vec{a}_{cn}) = m\vec{a}_{ct} + m\vec{a}_{cn}$$

$$\vec{v}_c = \vec{\omega} \times \vec{r}_c$$

$$\frac{d\vec{v}_c}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r}_c + \vec{\omega} \times \frac{d\vec{r}_c}{dt} = \underbrace{\vec{\alpha} \times \vec{r}_c}_{\vec{a}_{ct}} + \underbrace{\vec{\omega} \times \vec{v}_c}_{\vec{a}_{cn}}$$

normal acceleration:

$$\vec{a}_{cn} = \vec{\omega} \times \vec{v}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r}_c) = (\vec{\omega} \cdot \vec{r}_c) \cdot \vec{\omega} - (\vec{\omega}\vec{\omega}) \cdot \vec{r}_c = *$$

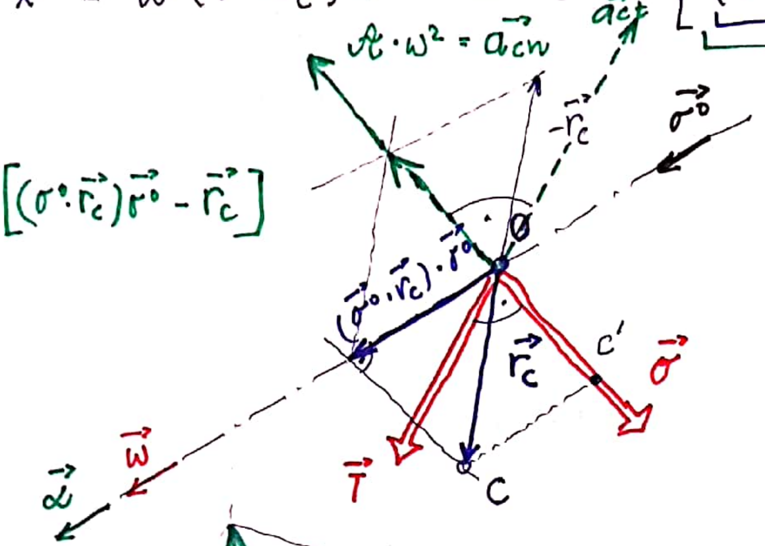
multiplication of 3 vectors

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\vec{c})\vec{b} - (\vec{a}\vec{b})\vec{c}}$$

we can use: $\vec{\omega} = \omega \vec{o}^{\,0}$

$$* = \omega^2 (\vec{o}^{\,0} \cdot \vec{r}_c) \cdot \vec{o}^{\,0} - \omega^2 \vec{r}_c = \underbrace{\left[\underline{(\vec{o}^{\,0} \cdot \vec{r}_c) \cdot \vec{o}^{\,0}} - \vec{r}_c \right]}_{A} \cdot \omega^2 = \vec{a}_{cn}$$

$A = \left[(\vec{o}^{\,0} \cdot \vec{r}_c)\vec{o}^{\,0} - \vec{r}_c\right]$



$\vec{a}_c = \vec{a}_{cn} + \vec{a}_{ct}$

$a_{ct} \perp a_{cn}$

$a_{ct} = \alpha \cdot r_c$

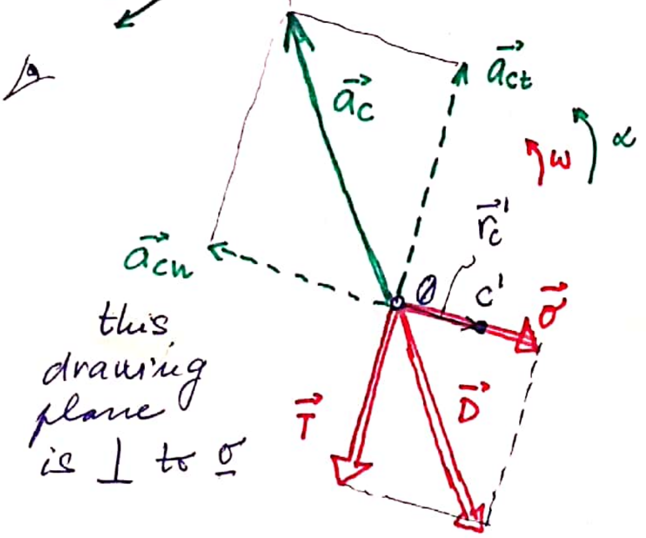
$\vec{D} = -\vec{a}_c \cdot m$

$\vec{D} = -\vec{a}_{cn} \cdot m - \vec{a}_{ct} \cdot m$

$\vec{D} = \vec{O} + \vec{T}$

$\vec{O} = -\vec{a}_{cn} \cdot m$

$\vec{T} = -a_{ct} \cdot m$



$o, \vec{r}_c, \vec{O}$ ... lies in one plane