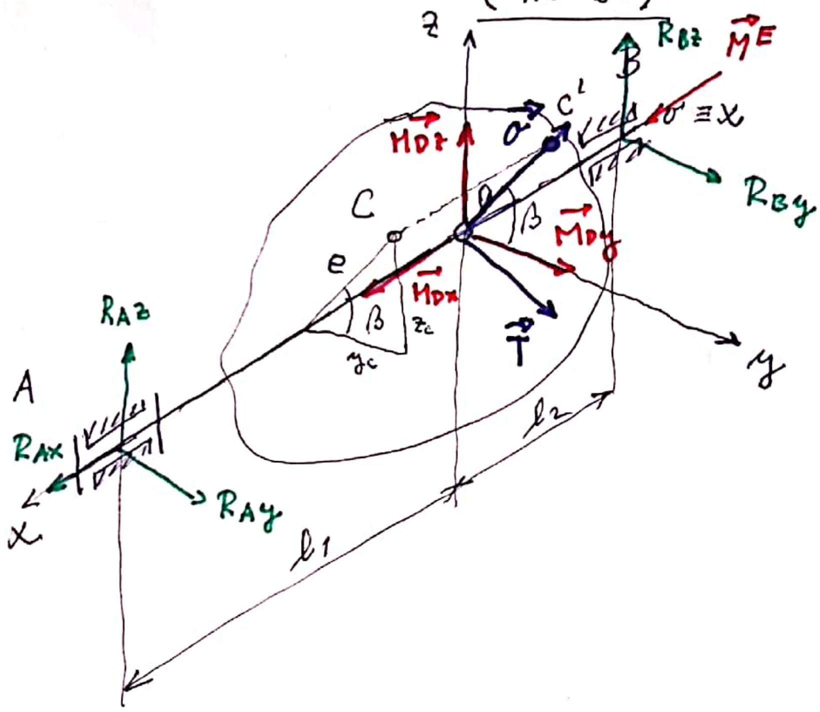
# FREE BODY DIAGRAM OF ROTATING BODY (IN 3D)






Given:

$C[x_c, y_c, z_c]$ ... position of center of mass of body

m ... mass of body

$J_x$ ... moment of inertia about x-axis

$D_{xy}, D_{xz}$ ... cross moments of inertia

$l_1, l_2$ ... dimensions

Task:
Calculate dynamical forces (real time) in joints A and B.

---

in free body diagram:

Action ~~forces~~ effects : $\vec{M}^E$

Reactional forces: $R_{Ax}, R_{Ay}, R_{Az}, R_{By}, R_{Bz}$

Dynamical forces: $\vec{D} = \vec{O} + \vec{T}$

$\vec{M}_D = \vec{M}_{Dx} + \vec{M}_{Dy} + \vec{M}_{Dz}$

C' ... projection of center of mass to plane (y, z) of coordinate system

---

Equations of dynamical equilibrium:

(1) $x: \; R_{Ax} = 0$

(2) $y: \; R_{Ay} + R_{By} + O \cdot \cos\beta + T \cdot \sin\beta = 0$

(3) $z: \; R_{Az} + R_{Bz} + O \cdot \sin\beta - T \cos\beta = 0$

(4) $\overset{\frown}{x}: \; M_{Dx} + M^E = 0$

(5) $\overset{\frown}{y}: \; -R_{Az} \cdot l_1 + R_{Bz}\, l_2 + M_{Dy} = 0$

(6) $\overset{\frown}{z}: \; R_{Ay} \cdot l_1 - R_{By}\, l_2 + M_{Dz} = 0$

---

Additional eqns:

(7) $\operatorname{tg}\beta = \frac{z_c}{y_c} \Rightarrow \beta = \dots$

(8) $O = m e \omega^2$

(9) $T = m e \alpha$

(10) $e = \sqrt{y_c^2 + z_c^2}$

(11) $M_{Dx} = -J_x \cdot \alpha$

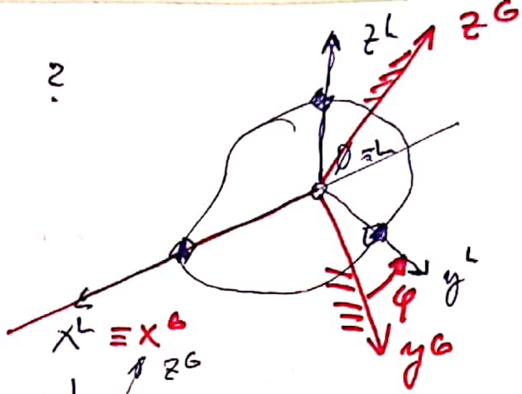
(12) $M_{Dy} = \alpha D_{xy} - \omega^2 D_{xz}$

(13) $M_{Dz} = \alpha D_{xz} + \omega^2 D_{xy}$

(14) $\alpha = \frac{d\omega}{dt}$ ; (15) $\omega = \frac{d\varphi}{dt}$

Finally: $R_{Ax}, R_{Ay}, R_{Az}, R_{By}, R_{Bz}$ ?

condition: $M^E = 0$



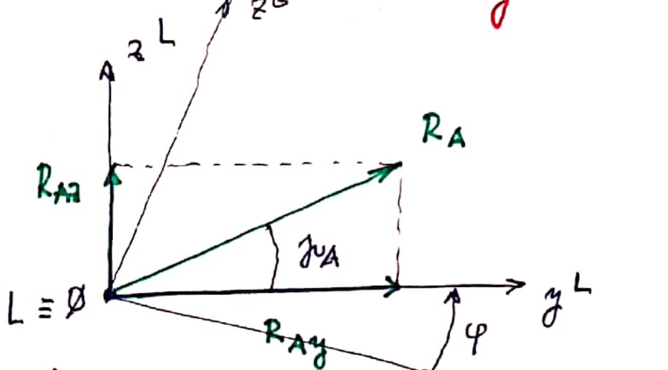

(1) $R_{Ax} = 0$

$R_{Ay} = f_1(\omega, \alpha, \dots)$

$R_{Az} = f_2(\omega, \alpha, \dots)$

$R_A = \sqrt{R_{Ay}^2 + R_{Az}^2}$

$\operatorname{tg} \gamma_A = \dfrac{R_{Az}}{R_{Ay}} \Rightarrow \gamma_A = \dots$




in coordinate system $(0, x, y, z)$ $R_A$ is constant.

local coordinate system connected with body

$(0, x^G, y^G, z^G)$ ... global coordinate system connected with ground

$R^G_{Ay} = R_A \cdot \cos(\gamma_A + \varphi)$

$R^G_{Az} = R_A \cdot \sin(\gamma_A + \varphi)$