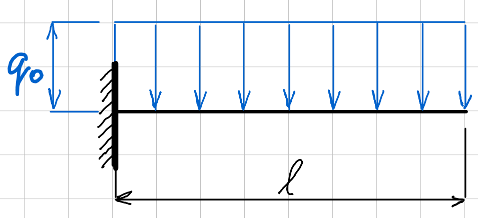
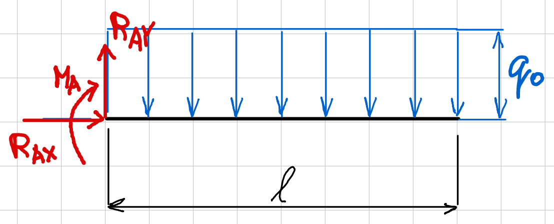
PR4

Dáno: q_0, ℓ

Určit: Vnější (R_{AX}, R_{AY}, M_A) a vnitřní ($N(x), T(x), M(x)$) statické účinky vetknutého nosníku zatíženého konst. spoj. zatížením.



Uvolnění:



Rovnice rovnováhy:

$$(x):\quad R_{AX} = 0 \qquad (1)$$

$$(y):\quad R_{AY} - q_0 \cdot \ell = 0 \qquad (2)$$

$$(\overset{\curvearrowleft}{M_A}):\quad -q_0 \cdot \ell \cdot \frac{\ell}{2} - M_A = 0 \qquad (3)$$

Výpočet reakcí:

$$(2) \rightarrow \boxed{R_{AY} = q_0 \cdot \ell}$$

$$(3) \rightarrow \boxed{M_A = -\frac{q_0 \cdot \ell^2}{2}}$$

Průběhy vnitřních statických účinků (VSÚ):

Schwedlerovy věty: 1. $q(x) = -\frac{dT}{dx}$

2. $T(x) = \frac{dM}{dx}$

$$dT = -q(x)\,dx$$

$$\int_{R_{AY}}^{T(x)} dT = \int_0^x -q_0\,dx$$

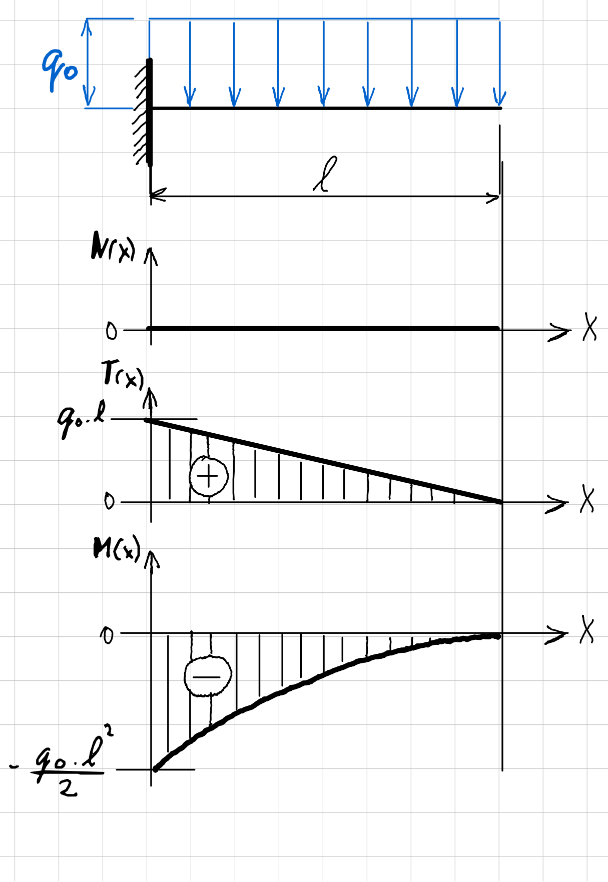
$$T(x) = R_{AY} - q_0 \cdot x = q_0 \cdot \ell - q_0 x = q_0(\ell - x)$$

$$dM = T(x)\,dx$$

$$\int_{M_A}^{M(x)} dM = \int_0^x q_0(\ell - x)\,dx$$

$$M(x) = M_A + q_0\left(\ell x - \frac{x^2}{2}\right) = -\frac{q_0 \cdot \ell^2}{2} + q_0\left(\ell x - \frac{x^2}{2}\right)$$

Grafy průběhů $N(x)$, $T(x)$, $M(x)$:

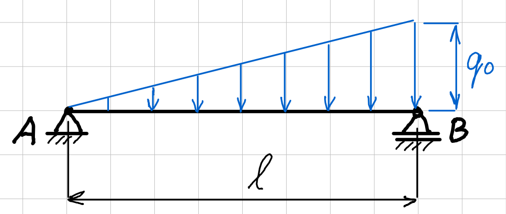


$N(x) = 0$

$T(x) = q_0(\ell - x)$

$M(x) = -\frac{q_0 \cdot \ell^2}{2} + q_0\left(\ell x - \frac{x^2}{2}\right)$

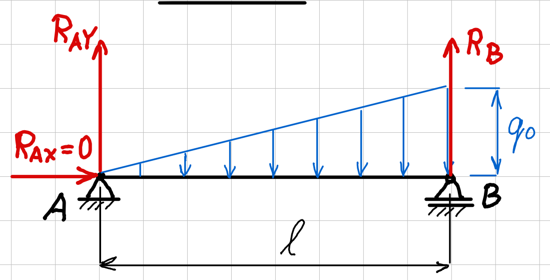
(PR5)



Dáno: q_0, ℓ

Určit: Vnější (R_{AX}, R_{AY}, R_B) a vnitřní ($N(x), T(x), M(x)$) statické účinky nosníku na dvou podporách zatíženého lineárně rostoucím spoj. zat.

Uvolnění:



Rovnice rovnováhy:

(x): $R_{Ax} = 0$ (1)

(y): $R_A + R_B - \frac{1}{2} q_0 \ell = 0$ (2)

(M_A): $R_B \cdot \ell - \frac{1}{2} q_0 \ell \cdot \frac{2}{3}\ell = 0$ (3)

Výpočet reakcí:

(3) → $\boxed{R_B = \frac{1}{3} q_0 \ell}$

(2) → $\boxed{R_A = \frac{1}{2} q_0 \ell - \frac{1}{3} q_0 \ell = \frac{1}{6} q_0 \ell}$

Průběhy vnitřních statických účinků (VSÚ):

Nejdřív musíme vyjádřit spoj. zat. jako funkci $q(x)$.

$$q(x) = \frac{q_0}{\ell} x$$

Schwedlerovy věty: 1. $q(x) = -\frac{dT}{dx}$ → $dT = -q(x)dx$ (4)

2. $T(x) = \frac{dM}{dx}$ → $dM = T(x)dx$ (5)

(4) → $\int_{R_{AY}}^{T(x)} dT = \int_0^x -\frac{q_0}{\ell} x \, dx$

$T(x) - R_{AY} = -\frac{q_0 x^2}{2\ell}$ → $\boxed{T(x) = \frac{1}{6} q_0 \ell - \frac{q_0 x^2}{2\ell}}$

(5) → $\int_0^{M(x)} dM = \int_0^x \left(\frac{1}{6} q_0 \ell - \frac{q_0 x^2}{2\ell}\right) dx$ → $\boxed{M(x) = \frac{1}{6} q_0 \ell x - \frac{1}{6} \frac{q_0}{\ell} x^3}$

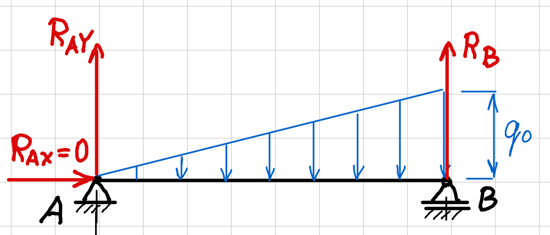
$T(0) = \frac{1}{6} q_0 \ell$, $T(\ell) = \frac{1}{6} q_0 \ell - \frac{1}{2} q_0 \ell = -\frac{1}{3} q_0 \ell$

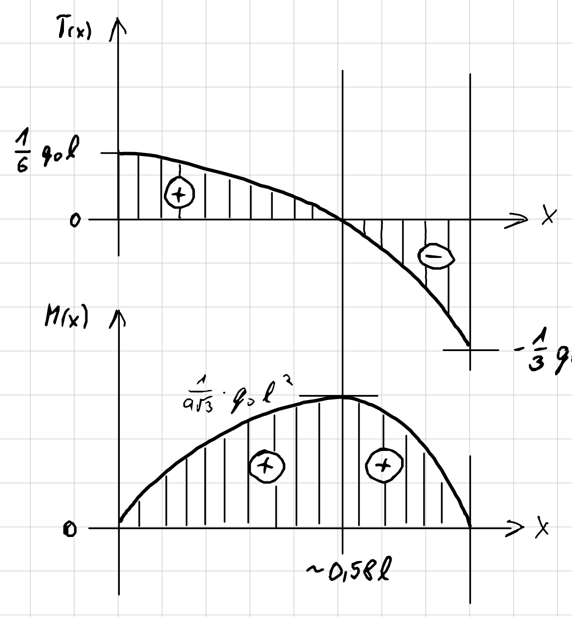
$M(0) = 0$, $M(\ell) = \frac{1}{6} q_0 \ell^2 - \frac{1}{6} q_0 \ell^2 = 0$

Grafy průběhů $N(x)$, $T(x)$, $M(x)$:

$\boxed{T(x) = \frac{1}{6} q_0 \ell - \frac{q_0 x^2}{2\ell}}$

$\boxed{M(x) = \frac{1}{6} q_0 \ell x - \frac{1}{6} \frac{q_0}{\ell} x^3}$





Výpočet M_{max}:

$\frac{dM}{dx} = T(x) = 0$

$\frac{1}{6} \cancel{q_0} \ell - \frac{\cancel{q_0} x_{max}^2}{2\ell} = 0$

$\frac{1}{3}\ell^2 = x_{max}^2$ → $x_{max} = \frac{1}{\sqrt{3}}\ell \doteq 0{,}58\ell$

$M_{max} = \frac{1}{6} q_0 \ell \frac{1}{\sqrt{3}}\ell - \frac{1}{6} \frac{q_0}{\ell} \frac{\ell^3}{3^{\frac{3}{2}}}$

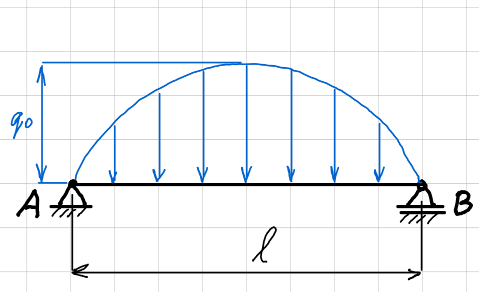
$M_{max} = \frac{1}{6\sqrt{3}} q_0 \ell^2 - \frac{1}{6\sqrt{3}^3} q_0 \ell^2$ *

$M_{max} = \frac{1}{9\sqrt{3}} \cdot q_0 \ell^2$

* $\sqrt{3}^3 = 3^{\frac{3}{2}} = \sqrt{3^3} = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

$\frac{1}{6\sqrt{3}} - \frac{1}{6 \cdot 3 \cdot \sqrt{3}} = \frac{2}{3} \cdot \frac{1}{6\sqrt{3}} = \frac{1}{9\sqrt{3}}$

PŘ6



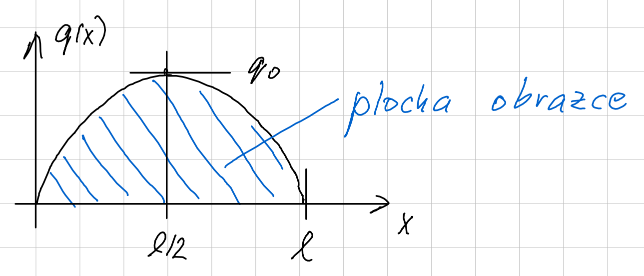
Dáno: q_0, ℓ

Určit: Vnější (R_{AX}, R_{AY}, R_B) a vnitřní $(N(x), T(x), M(x))$ statické účinky nosníku na dvou podporách zatíženého spoj. zat. ve tvaru sinusovky.

Vyjádříme spojité zatížení jako funkci $q(x)$.

$q(0) = 0, \quad q(\ell) = 0, \quad q(\ell/2) = q_0$

$$\boxed{q(x) = q_0 \cdot \sin\left(\frac{\pi}{\ell} x\right)}$$



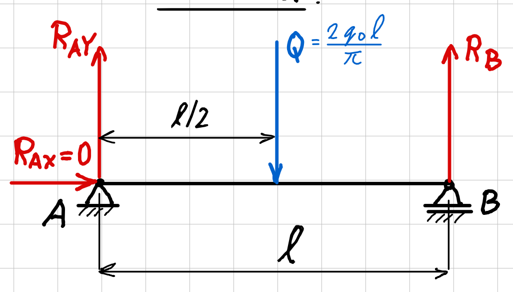
Spoj. zat. $q(x)$ lze při výpočtu reakcí nahradit silou Q, ta je rovna ploše obrazce. Plochu vypočítáme pomocí integrálu:

$$Q = \int_0^{\ell} q_0 \cdot \sin\left(\frac{\pi}{\ell}x\right) dx \overset{*}{=} -\frac{q_0 \ell}{\pi}\left[\cos\left(\frac{\pi}{\ell}x\right)\right]_0^{\ell} = -\frac{q_0\ell}{\pi}\underbrace{\left(\underbrace{\cos(\pi)}_{-1} - \underbrace{\cos(0)}_{1}\right)}_{-1-1=-2}$$

$$Q = \frac{2 q_0 \ell}{\pi}$$

$$\left(\begin{aligned} &\int_0^x a \cdot \sin(bx)\,dx \overset{*}{=} -a \cdot \frac{1}{b}\left[\cos(bx)\right]_0^x \\ &\int_0^x a \cdot \cos(bx)\,dx = a \cdot \frac{1}{b}\left[\sin(bx)\right]_0^x \end{aligned}\right)$$

Uvolnění:



Rovnice rovnováhy:

(x): $\quad R_{AX} = 0 \qquad (1)$

(y): $\quad R_{AY} + R_B - \frac{2q_0\ell}{\pi} = 0 \qquad (2)$

(M_A): $\quad R_B \cdot \ell - \frac{2q_0\ell}{\pi} \cdot \frac{\ell}{2} = 0 \qquad (3)$

Výpočet reakcí:

$(3) \rightarrow \quad \boxed{R_B = \frac{q_0 \ell}{\pi}}$

$(2) \rightarrow \quad \boxed{R_{AY} = \frac{2q_0\ell}{\pi} - R_B = \frac{q_0\ell}{\pi}}$

I ze symetrie plyne, že se síla Q rovnoměrně rozloží, tedy

$R_{AY} = R_B = \frac{Q}{2} = \frac{q_0\ell}{\pi}$

Průběhy vnitřních statických účinků (VSÚ):

$q(x) = q_0 \cdot \sin\left(\frac{\pi}{\ell}x\right)$

Schwedlerovy věty: 1. $q(x) = -\frac{dT}{dx} \quad \rightarrow \quad dT = -q(x)dx \qquad (4)$

2. $T(x) = \frac{dM}{dx} \quad \rightarrow \quad dM = T(x)dx \qquad (5)$

$(4) \rightarrow \quad \int_{R_{AY}}^{T(x)} dT = \int_0^x -q_0 \cdot \sin\left(\frac{\pi}{\ell}x\right)dx$

$$T(x) - R_{AY} = \frac{q_0\ell}{\pi}\left[\cos\left(\frac{\pi}{\ell}x\right)\right]_0^x = \frac{q_0\ell}{\pi}\left(\cos\left(\frac{\pi}{\ell}x\right) - \underbrace{\cos(0)}_{1}\right)$$

$$T(x) = \frac{q_0\ell}{\pi} + \frac{q_0\ell}{\pi}\cos\left(\frac{\pi}{\ell}x\right) - \frac{q_0\ell}{\pi}$$

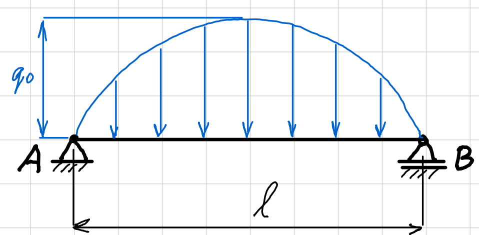
$$\boxed{T(x) = \frac{q_0\ell}{\pi}\cos\left(\frac{\pi}{\ell}x\right)}$$

$(5) \rightarrow \quad \int_0^{M(x)} dM = \int_0^x \frac{q_0\ell}{\pi}\cos\left(\frac{\pi}{\ell}x\right)dx$

$$M(x) = \frac{q_0\ell}{\pi} \cdot \frac{\ell}{\pi} \cdot \left[\sin\left(\frac{\pi}{\ell}x\right)\right]_0^x$$

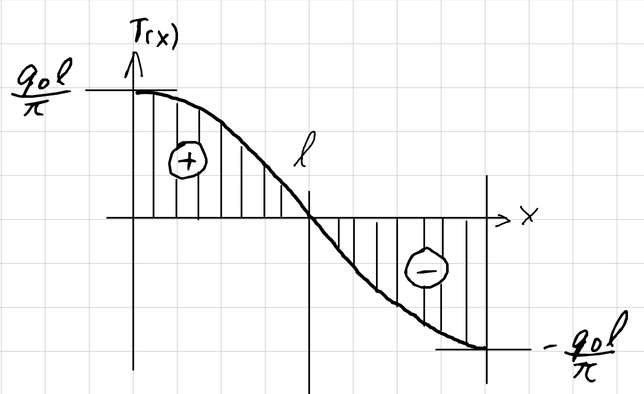
$$\boxed{M(x) = q_0\left(\frac{\ell}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{\ell}x\right)}$$

Grafy průběhů $N(x), T(x), M(x)$:



$$\boxed{T(x) = \frac{q_0\ell}{\pi}\cos\left(\frac{\pi}{\ell}x\right)}$$

$$\boxed{M(x) = q_0\left(\frac{\ell}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{\ell}x\right)}$$

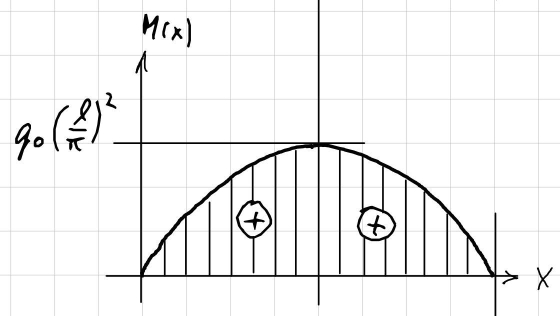


$T(0) = \frac{q_0\ell}{\pi} \cdot \cos(0) = \frac{q_0\ell}{\pi}$

$T(\ell) = \frac{q_0\ell}{\pi}\cos(\pi) = -\frac{q_0\ell}{\pi}$

$M(0) = q_0\left(\frac{\ell}{\pi}\right)^2 \cdot \sin(0) = 0$

$M(\ell) = q_0\left(\frac{\ell}{\pi}\right)^2 \cdot \sin(\pi) = 0$



$M_{max} = M\left(x = \frac{\ell}{2}\right)$

$M_{max} = q_0\left(\frac{\ell}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{2}\right) = q_0\left(\frac{\ell}{\pi}\right)^2$